Section 5.5
Graphing Systems of Inequalities

The solution to a system of equations is the point where the lines meet. The solution to a system of inequalities is not a point but a region surrounded by the lines the system of inequalities represents.

Example: Find the solution to the inequalities \( x + y \geq 3 \) and \( y \leq x - 2 \)

First, plot the inequalities using the same method used for equations:

The slope and \( y \)-intercept of the first inequality:
\[
\begin{align*}
x + y & \geq 3 \\
x - x + y & \geq -x + 3 \\
y & \geq -x + 3
\end{align*}
\]

The slope (coefficient of \( x \)) is \(-1\) and the \( y \)-intercept is \(3\).

The slope of the second inequality is \(1\) (coefficient of \( x \)) and the \( y \)-intercept is \(-2\). Both lines are plotted.

To find the region of the answer, look at the direction of the \( y \) inequalities only. In the first inequality, \( y \) is “greater than”; therefore, the solution is above (greater). The second inequality is “less than”, thus the solution must be below the line. The graphs below show this for each line separately and then a third graph shows the solution (both combined).

Plotted separately

\( y \geq -x + 3 \) \hspace{1cm} \( y \leq x - 2 \)

Solution region for both inequalities together. Every point in the solution region will work for both inequalities.
Solution regions for pairs of inequalities (positive slope and negative slope):

**Examples:**

- \( y > \)
  - \( y > \)          
  - \( y > \)
  - \( y > \)

- Both pos. and neg. slopes “greater than”
- Both pos. and neg. slopes “less than”
- Neg. slope “greater than” pos. slope “less than”
- Pos. slope “greater than” neg. slope “less than”

**Example:** Find the solution region for \( x < -5 \) and \( y > -2 \)

The first inequality is to the left of a vertical line (\( x \) is “less” to the left.)

The second inequality is above a horizontal line (\( y \) is “greater” over the horizontal.)

Because the inequalities are \(<\) and \(>\), dashed lines apply.

**Example:** Find the solution region for \( y \leq 3 \) and \( y \geq 2x - 1 \)

Because both lines are \(\leq\) and \(\geq\), solid lines apply.
Example: Find the solution area to the four inequalities:

\[
\begin{align*}
&x < 2 \quad \text{(dashed line)} \\
&y \leq 3 \quad \text{(solid line)} \\
&y \geq x - 4 \quad \text{(solid line)} \\
&y < 3x + 1 \quad \text{(dashed line)}
\end{align*}
\]

Practice:
Find the solution area.

1. \(y \geq x + 5\)  
   \(y \geq -x + 3\)
2. \(x + y > 8\)  
   \(x - y > 6\)
3. \(x + y < 1\)  
   \(y + 4x < 6\)
4. \(y < -x - 2\)  
   \(3x + y \leq -5\)
5. \(3y \geq 2x + 12\)  
   \(-5x + 6y + 14 \leq 0\)
6. \(4y > 5x + 8\)  
   \(-3x + 5y \leq 2\)
7. \(-5x < 3y - 15\)  
   \(2y < 3x - 4\)
8. \(5x + 4y + 12 \leq 0\)  
   \(4y - 3x + 8 \leq 0\)
9. \(4y \geq 7x + 11\)  
   \(4y \geq -2x - 9\)
10. \(8x > 7y - 12\)  
    \(3x + y + 13 > 0\)
11. \(3y \leq 9x + 13.5\)  
    \(y < 7\)
12. \(5y \leq 2x - 8\)  
    \(7x + 6y \leq -6\)
13. \(3y \leq 4x + 6\)  
    \(-2x + 3y + 15 \geq 0\)
14. \(7x \leq 4y + 1\)  
    \(4x + 2y > 15\)
15. \(-9x > -4y - 16\)  
    \(4y < 9x - 8\)
16. \(6x + 3y + 17 \leq 0\)  
    \(8y + 5x + 14 \leq 0\)
17. \(4y \geq 9x + 24\)  
    \(3y \geq -5x + 15\)
18. \(5y > 7x - 21\)  
    \(7x + 5y + 10 > 0\)
19. \(2y < 7x + 11\)  
    \(y \leq 5\)
20. \(3y \leq 8x - 12\)  
    \(3x + 5y \leq 12\)
21. \(2y \geq 3x\)  
    \(-2x + 3y - 6 \leq 0\)
22. \(3y > 7x + 12\)  
    \(3x + 4y \leq 9\)
23. \(-3x > -3y - 6\)  
    \(y < 5x - 5\)
24. \(5x + 2y + 7 \geq 0\)  
    \(7y + 5x + 14 \geq 0\)
25. \(2y \leq 3x + 4\)  
    \(4y \leq -3x + 3\)
26. \(2y > 5x - 7\)  
    \(x + 2y + 6 > 0\)
27. \(3y < 5x + 8\)  
    \(y \geq -6\)

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