Section 5.2
Systems of Equations: Solve by Substitution

THE SUBSTITUTION METHOD

Using the second example from section 5.1:

\[ y = 2x - 1 \quad y = \frac{2}{3}x + 2 \]

because \[ y = y \]
then \[ 2x - 1 = \frac{2}{3}x + 2 \]

When we pair these two expressions, we get rid of \( y \) and are left with one \( x \) on each side. We then combine “like terms” and solve for \( x \).

Solving a rational equation (an equation with fractions) is easier if we eliminate the fractions and turn them into integers. We do this by multiplying each term of the equation by 3 (because the 3 denominator is what makes it a fraction).

\[ (3)(2x) - (3)(1) = (3)\left(\frac{2}{3}x\right) + (3)(2) \]

The result is that we exchange canceling the 3 denominator for a larger equation—of the same value—overall. The new equation is now:

\[ 6x - 3 = 2x + 6 \]

Combine like terms
\[ 6x - 2x = 6 + 3 \]
\[ 4x = 9 \]
\[ x = \frac{9}{4} = 2.25 \]

The graphical solution in section 5.1 estimated \( x = 2.3 \), but \( x = 2.25 \) is accurate and exact.

To find the \( y \) value of the equation, go back to either of the two original equations, substitute the value of \( x \), and get the value for \( y \):

\[ y = 2(2.25) - 1 \]
\[ y = 4.5 - 1 \]
\[ y = 3.5 \]

The point where the lines cross is (2.25, 3.5). Any system of equations can be solved by substitution.

Example: Solve the system by substitution

\[ y = 2x + 7 \quad y = 2x + 4 \]

Because \( y = y \), substitute

\[ 2x + 7 = 2x + 4 \]
\[ 2x - 2x = -7 + 4 \]
\[ 0 = -3 \]

Because 0 is not equal to –3, the lines will not meet and there is no solution to the system: The lines are parallel. If the solution had been a true statement, like –3 = –3, then there is only one solution (all the points are at the intersection) and both lines are the same (identity property).
Example: Solve the system by substitution

\[
\begin{align*}
2x - 3y &= 10 \\
x + y &= 2
\end{align*}
\]

To substitute, first solve one of the equations in terms of \(x\) or \(y\). Solving for \(x\), the second equation becomes:

\[x = -y + 2\]

Substituting \((-y + 2)\) for \(x\) into the first equation:

\[2(-y + 2) - 3y = 10\]

Doing this gives us an equation without \(x\).

Solving for \(y\):

\[
\begin{align*}
2(-y + 2) - 3y &= 10 \\
-2y + 4 - 3y &= 10 \\
-5y + 4 &= 10 - 4 \\
-5y &= 6 \\
y &= \frac{6}{-5} = -1.2
\end{align*}
\]

Because \(x = -y + 2\) and \(y = -1.2\),

by substitution, then

\[
\begin{align*}
x &= -(-1.2) + 2 \\
x &= 1.2 + 2 \\
x &= 3.2
\end{align*}
\]

The solution to the system is point \((3.2, -1.2)\)

Practice:

Solve each system by substitution.

1. \[
\begin{align*}
2y &= 3x + 5 \\
y &= x + 3
\end{align*}
\]

9. \[
\begin{align*}
y &= 3x + 1 \\
y &= -x + 6
\end{align*}
\]

17. \[
\begin{align*}
2y &= 2x + 10 \\
y &= -5x + 11
\end{align*}
\]

2. \[
\begin{align*}
y &= 4x - 5 \\
x &= 2y - 3 \\
x + 2y + 6 &= 0 \\
3x + y + 5 &= 0
\end{align*}
\]

10. \[
\begin{align*}
x &= 2y - 3 \\
y &= 2x + 3
\end{align*}
\]

18. \[
\begin{align*}
3y &= 4x - 4 \\
x + y + 6 &= 0
\end{align*}
\]

3. \[
\begin{align*}
3y &= 5x \\
y &= -5 \\
x + y &= -3
\end{align*}
\]

11. \[
\begin{align*}
y &= 2x + 3 \\
y &= 6
\end{align*}
\]

19. \[
\begin{align*}
6y &= x + 9 \\
y &= 7
\end{align*}
\]

4. \[
\begin{align*}
y &= -x - 7 \\
x + y &= -3 \\
x &= y + 7 \\
x + 4y &= -12
\end{align*}
\]

12. \[
\begin{align*}
7y &= x - 10 \\
x &= y + 13 \\
3x + y &= 4 \\
3x + y &= 9
\end{align*}
\]

13. \[
\begin{align*}
5y &= x + 1 \\
x &= y + 13 \\
x &= y + 13 \\
3x + y &= 4
\end{align*}
\]

14. \[
\begin{align*}
x &= y + 13 \\
x &= y + 13 \\
x &= y + 13
\end{align*}
\]

15. \[
\begin{align*}
-2x &= -y - 6 \\
y &= x - 5 \\
y &= x - 5
\end{align*}
\]

16. \[
\begin{align*}
3x + 2y + 5 &= 0 \\
y + 2x + 9 &= 0 \\
3y + x + 5 &= 0
\end{align*}
\]

17. \[
\begin{align*}
2y &= 2x + 10 \\
y &= -5x + 11
\end{align*}
\]

18. \[
\begin{align*}
3y &= 4x - 4 \\
x + y + 6 &= 0
\end{align*}
\]

19. \[
\begin{align*}
6y &= x + 9 \\
y &= 7
\end{align*}
\]

20. \[
\begin{align*}
2x + 3y &= 5
\end{align*}
\]

21. \[
\begin{align*}
3x + y &= 9
\end{align*}
\]

22. \[
\begin{align*}
3y &= 5x + 7 \\
-3x + 9y + 3 &= 0
\end{align*}
\]

23. \[
\begin{align*}
-4x &= -2y - 5 \\
y &= 3x - 2
\end{align*}
\]

24. \[
\begin{align*}
2x + 3y + 7 &= 0 \\
3y + x + 5 &= 0
\end{align*}
\]